JBB-003-1163005 Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

December - 2019

Mathematics : CMT - 3004

(Discrete Mathematics) (Old & New Course)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all the questions.
(2) There are 5 questions.
(3) All questions carry equal marks.

1 Answer any **seven** questions : **7×2=14**

(a) Define: Congruence relation on semigroups.

(b) Let $f:(S,*')\to(T,*)$ be a homomorphism of semigroups. Prove that the image of *f* is a subsemigroup of (*T*, *',).

(c) Give an example of lattice which is modular but not distributive.

(d) Prove that any finite lattice is bounded.

(e) Define : Context free grammar and Context free language.

(f) Find regular expression of language of all string of length eight or less.

(g) Define: Toutology with an example.

(h) Define: Existential quantification and Existential quantifier.

(i) Explain: Channel and Noise.

(j) Decide which codeword was transmitted if the received codeword is 1100111 from Hamming code of length seven with three parity bits.

2 Answer any **two** : **2×7=14**

(a) State and prove Fundamental theorem of homomorphism of semigroups.

(b) Let R be a relation defined on a non-empty set A. Then prove that the transitive closure of R equals $\bigcup_{n=1}^{\infty} R^n$.

(c) Let G be a group. Let R be a congruence relation on G. Prove that there exist a normal subgroup N of G such that for any $a,b\in G, aRb$ if and only if $ab^{-1}\in N$.

3 Answer the following : **2×7=14**

(a) Let $(L, \leq)$ be a lattice. Suppose that $(L, \leq)$ is modular. Prove that $(L, \leq)$ is distributive iff L does not contain any sublattice which is isomorphic to the pentagon lattice.

(b) Let $n > 1$ then prove that $(D_n, \leq_{div})$ is complemented iff n is the product of distinct primes.

OR

(a) Let $(L, \leq)$ be a lattice. Then prove that $(L, \leq)$ is distributive iff for all $a, b, c \in L$,

$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$

(b) Let V be non zero vector space defined on field F then $(L(V), \subseteq)$ is distributive iff $\dim_F V = 1$.

4 Answer any **two** : **2×7=14**

(a) Find Context free grammars of following languages :

(1) Language of non-palindromes

(2) $L = \left\{ x \in \{0,1\}^* \middle| n_0(x) = n_1(x) \right\}$

(b) Show that for any NFA $M = (Q, \Sigma, q_0, A, \delta)$ accepting a language $L \subseteq \Sigma^*$, there is an FA $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$, that also accepts L.

(c) Draw FAs corresponding to following regular expressions.

(1) $(11 + 10)^*$

(2) $(0 + 1)^* (1 + 00) (0 + 1)^*$

5 Answer any **two** : **2×7=14**

(a) Using indirect proof technique show that for all x, $x^2 + 1$ is odd then x is even.

(b) Let p and q be two statements then prove that

(1) $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$

(2) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

(c) Explain Hamming code and find Hamming code of length seven with three parity bits.

(d) Show that a binary code C can correct up to k errors in any codeword if and only if $d(C) \geq 2k + 1$.